

Feel free to work with other students, but make sure you write up the homework and code on your own (no copying homework *or* code; no pair programming). Feel free to ask students or instructors for help debugging code or whatever else, though.

*Note:* You need to create a Github account for submission of the coding part of the homework. Please create a repository on Github to hold all your code and include your Github account username as part of the answer to the coding problems.

**1 (Covariance. (Lecture 1 page 17))** The covariance between two random variables  $X$  and  $Y$  is defined as:

$$\text{cov}[X, Y] = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])].$$

Prove that

$$\text{cov}[X, Y] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y].$$

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**2 (Correlation. (Lecture 1 page 18))** The correlation between two random variables  $X$  and  $Y$  is defined as:

$$\text{corr}[X, Y] = \frac{\text{cov}[X, Y]}{\sqrt{\text{var}[X]\text{var}[Y]}}.$$

Prove that

- (a)  $-1 \leq \text{corr}[X, Y] \leq 1$ ;
- (b)  $\text{corr}[X, Y] = 1$  if and only if  $Y = aX + b$  for some parameters  $a \neq 0$  and  $b$ .

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**3 (Parametrization. (Lecture 1 page 50))** Let  $\alpha(t)$  be a parametrized curve which does not pass through the origin. If  $\alpha(t_0)$  is the point of the trace of  $\alpha$  closest to the origin and  $\alpha'(t_0) \neq 0$ , show that the position vector  $\alpha(t_0)$  is orthogonal to  $\alpha'(t_0)$ .

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**4 (Extra credit. (Lecture 1 page 52))** How to create a transformation from the data on some helix to the data of the instructors trajectory?

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**5 (Coding. (Lecture 1 page 54-70))** Please download the H-MOG dataset from: <http://www.cs.wm.edu/qyang/hmog.html> (see also Lecture 1 page 54). Please read through the data description and do some visualizations if you have time. If you have any visualization result, please email or print out to submit.

